A possible origin of superconducting currents in cosmic strings

Helder Chávez [‡] [*], Luis Masperi[✠] [†]

[‡] Centro Brasileiro de Pesquisas Físicas,
Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil

[✠] Centro Latinoamericano de Física,
Av. Venceslau Bráz 71 Fundos, 22290-140 Rio de Janeiro, Brazil

June 27, 2002

Abstract

The scattering and capture of right-handed neutrinos by an Abelian cosmic string in the $SO(10)$ grand unification model are considered. The scattering cross-section of neutrinos per unit length due to the interaction with the gauge and Higgs fields of the string is much larger in its scaling regime than in the friction one because of the larger infrared cutoff of the former.The probability of capture in a zero mode of the string accompanied by the emission of a gauge or Higgs boson shows a resonant peak for neutrino momentum of the order of its mass. Considering the decrease of number of strings per unit comoving volume in the scaling epoch the cosmological consequences of the superconducting strings formed in this regime will be much smaller than those which could be produced already in the friction one.

1 Introduction

It is possible that the early universe suffered a sequence of phase transitions breaking symmetries of the grand unification theories (GUT) and generating topological defects [1] like the cosmic strings.

[*]Fellow of Centro Latinoamericano de Física.

[†]On leave of absence from Centro Atómico Bariloche, San Carlos de Bariloche, Argentina. Author to whom correspondence should be addressed.

The Aharonov-Bohm effect has been described [2] for the interaction of fermions of charge e with the string gauge field when the ratio $\frac{e}{e_o}$ is half-integer, being e_o the charge of the Higgs field responsible for the breaking of the Abelian symmetry which generates the defect.

The corresponding cross-section diverges in the forward and backward directions due to the long range nature of the interaction which gives contributions even for wave packets passing at infinite distance from the string. However the unitarity of the S-matrix has been proved [3] and the cross-section can be taken as finite introducing a cutoff for the distance from the string.

This cutoff has the physical motivation in the correlation length between strings. At the beginning of the so called friction regime after their formation $\xi \simeq \frac{1}{\lambda\eta}$ where λ is the coupling constant of the Higgs potential which breaks the symmetry at the energy scale η. Afterwards, if the scaling regime is achieved $\xi \backsim t$ so that at any time strings represent the same fraction of the universe energy density.

The purpose of our work is twofold. On one side to include the effect of the string Higgs field in the scattering whose order of magnitude has been estimated [4] without considering the simultaneous interaction with the gauge field. On the other hand we wish to analyze the capture of fermions by the string to form superconducting currents if there are bound zero modes for the Dirac equation in the plane transverse to it [5] . For this last process the fermion capture has been considered [4] with the simultaneous emission of a Higgs particle, whereas we also include the alternative emission of a gauge boson.

The simplest fermionic candidate suitable for our analysis is the right-handed neutrino ν_R , which is a $SU(5)$ singlet in the representation **16** of $SO(10)$ and that acquires mass through a Majorana coupling with a $SU(5)$ singlet Higgs in the representation **126** of $SO(10)$, responsible for the breaking of the Abelian $\widetilde{U}(1)$ contained in the latter. Therefore ν_R is the only fermion which may form a zero mode when it is captured by the string generated at the breaking of $\widetilde{U}(1)$.

In Sect. 2 we describe the Abelian string in $SO(10)$ and the relevant fermionic field. In Sect. 3 the cross-section for scattering of Majorana neutrinos is calculated perturbatively in the approximation of large momentum both in the friction and scaling string regimes. Sect. 4 is devoted to the capture of fermion with the emission either of a Higgs or a gauge boson, finding in both cases a resonant peak for comparatively low momentum. Sect. 5 includes conclusions and cosmological implications of a larger influence of superconducting strings formed in the friction epoch.

2 SO(10) Abelian cosmic strings and Majorana fermions

The first GUT symmetry which contains an Abelian group $\widetilde{U}(1)$ additional to the electromagnetic one is $SO(10)$ which can be broken according to the scheme

$$SO(10) \underset{\mathbf{45}}{\longrightarrow} SU(5) \otimes \widetilde{U}(1) \underset{\mathbf{126}}{\longrightarrow} SU(5) \otimes Z_2 \underset{\mathbf{45}}{\longrightarrow} SU(3)_C \otimes SU(2)_L \otimes \widetilde{U}(1)_Y \otimes Z_2$$
$$\underset{\mathbf{10}}{\longrightarrow} SU(3)_C \otimes U(1)_{em} \otimes Z_2 \tag{1}$$

where the representations of the relevant Higgs fields are indicated. The expectation value of the Higgs field Φ in **126** breaks $\widetilde{U}(1)$ producing Abelian strings which are topologically stable because the conserved discrete symmetry Z_2 avoids their breaking by monopoles. The expectation value of Φ will be of the GUT order $\eta \backsim 10^{15} GeV$ and since its $\widetilde{U}(1)$ charge is 10 whereas that of ν_R denoted by ψ is 5 , a Majorana mass term coupling which violates lepton number is possible in the Lagrangian

$$\mathcal{L} = (\mathcal{D}_\mu \Phi)^\star (\mathcal{D}^\mu \Phi) - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4}\lambda \left(|\Phi|^2 - \eta^2\right)^2 + \psi^\dagger i \sigma^\mu \mathcal{D}_\mu \psi -$$
$$- \frac{1}{2}\left\{ig\psi^\dagger \Phi \psi^c + h.c.\right\} \ , \tag{2}$$

with $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$, $\mathcal{D}_\mu \Phi = (\partial_\mu - ie\mathcal{A}_\mu)\Phi$, $\mathcal{D}_\mu \psi = (\partial_\mu - \frac{1}{2}ie\mathcal{A}_\mu)\psi$, $\nu_R^c = i\sigma^2 \nu_R^\star$, $\sigma^\mu = (I, \sigma^i)$. In the broken symmetry vacuum $\Phi = \eta$ and $\mathcal{A}_\mu = 0$ there is a generation of masses $M_{\mathcal{H}} = \sqrt{\lambda}\eta$ and $M_{\mathcal{A}} = \sqrt{2}e\eta$ for the bosons and $M_o = g\eta$ for the fermion.

The string configuration in planar coordinates for unit winding number is

$$\Phi = \eta f\ (r)\ e^{i\varphi} \ \ , \ \ \mathcal{A}_\varphi = \frac{1}{e\,r}\, a\,(r) \tag{3}$$

with the behaviour $f\,(0) = a(0) = 0$, $f\,(\infty) = a(\infty) = 1$.

The free quantum Majorana field of right chirality [6] is

$$\psi\,(x) = \frac{1}{\sqrt{\mathcal{V}}} \sum_{\overrightarrow{p}} \frac{1}{\sqrt{2p_o}} [(c\,(\mathbf{p},+)\,e^{-ip.x} + c^\dagger\,(\mathbf{p},-)\,e^{ip.x})\sqrt{p_o + p}\,\chi\,(\overrightarrow{p},+) +$$
$$(c\,(\mathbf{p},-)\,e^{-ip.x} - c^\dagger\,(\mathbf{p},+)\,e^{ip.x})\sqrt{p_o - p}\,\chi\,(\mathbf{p},-)] \ , \tag{4}$$

where χ are the helicity eigenstates

$$\sigma.\mathbf{p}\,\chi\left(\mathbf{p},\pm\right)=\pm\,p\chi\left(\mathbf{p},\pm\right)\ , \tag{5}$$

and, having taken the finite normalization volume, the anticonmutation relation for the corresponding annihilation and creation operators is

$$\left\{c\left(\mathbf{p},\pm\right),c^{\dagger}\left(\mathbf{p}',\pm\right)\right\}=\delta_{\mathbf{p}\,,\mathbf{p}'}\ . \tag{6}$$

The phase between helicity states has been chosen to satisfy

$$\pm\sigma^{2}\chi^{\star}\left(\mathbf{p},\mp\right)=\chi\left(\mathbf{p},\pm\right)\ . \tag{7}$$

Convenient basis for a fermion moving in the $x\ y$ plane are

$$\chi\left(\mathbf{p},+\right)=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\qquad,\qquad\chi\left(\mathbf{p},-\right)=\frac{-i}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\ , \tag{8}$$

$$\chi\left(\mathbf{p}',+\right)=\frac{1}{\sqrt{2}}\begin{pmatrix}e^{i\theta/2}\\-e^{-i\theta/2}\end{pmatrix}\qquad,\qquad\chi\left(\mathbf{p}',-\right)=\frac{-i}{\sqrt{2}}\begin{pmatrix}e^{i\theta/2}\\e^{-i\theta/2}\end{pmatrix}\ , \tag{9}$$

respectively for the initial state when it comes from the positive x axis and the final one where it is scattered with angle θ .

3 Neutrino scattering by string in friction and scaling regimes

From Eq. (2) the interaction of ν_R of mass M_o with the string is given by

$$\begin{aligned}\mathcal{L}_{int}=&\frac{e}{2}\psi^{\dagger}\sigma^{\mu}\mathcal{A}_{\mu}\psi-\frac{M_o}{2}\psi^{\dagger}\sigma^{2}\psi^{\star}\left(1-e^{i\varphi}f\ (\rho)\right)\\&-\frac{M_o}{2}\psi^{T}\sigma^{2}\psi\left(1-e^{-i\varphi}f\ (\rho)\right)\ ,\end{aligned} \tag{10}$$

where, because of the terms of interaction with the Higgs field, the perturbative method will be applicable for $p>M_o$.

In the string rest frame, the cross-section due to an interaction time $\mathcal{T}$ will be

$$\sigma = \sum_{\text{final states}} \frac{\mathcal{V}}{\mathcal{T}} \frac{p_o}{p} \left| S_{f\ i} \right|^2 \ , \tag{11}$$

where the sum over final states includes momenta and helicities.

For elastic scattering, the S-matrix elements will receive three contributions

$$S_{f\ i} = S_{f\ i}^{(1)} + S_{f\ i}^{(2)} + S_{f\ i}^{(3)} \ , \tag{12}$$

where the first corresponds to the interaction with the gauge field and the others to those with Higgs field. For the perturbative evaluation of Eq.(12) we will approximate the behaviour of these bosonic classical field Eq.(3) of the string whose core radius is $R \backsim \eta^{-1}$ as

$$f(r) = a(r) = 0 \ , \qquad r < R \tag{13}$$

$$f(r) = a(r) = 1 \ , \qquad r > R \ . \tag{14}$$

The gauge field contribution to the scattering from positive to positive helicity fermion in first order perturbation considering Eqs.(4,8,9) gives

$$S_{+,+}^{(1)} = i \int d^4x \ \left\langle \mathbf{p}', + \left| \frac{e}{2} \psi^{\dagger} \sigma^{\mu} \mathcal{A}_{\mu} \psi \right| \mathbf{p}, + \right\rangle$$

$$= \frac{ieL}{2\mathcal{V}} \sqrt{\frac{(p_o' + p')(p_o + p)}{2p_o' \, 2p_o}} 2\pi \delta \left(p_o' - p_o \right) \ A_{+,+} \ , \tag{15}$$

for a length L of the string and where

$$A_{+,+} = \int dr d\varphi \ e^{-i\mathbf{Q}.\mathbf{r}} \ \frac{ia(r)}{2e} \left(e^{i\theta/2 + i\varphi} - e^{-i\theta/2 - i\varphi} \right) \ . \tag{16}$$

Using the expansion in plane waves

$$e^{-i\mathbf{Q}.\mathbf{r}} = \sum_{l=-\infty}^{\infty} (-i)^l \ J_l \left(Q \, r \right) \ e^{-il(\varphi - \beta)} \ , \tag{17}$$

where β is the angle between the momentum transfer $\mathbf{Q} = \mathbf{p}' - \mathbf{p}$ with x axis, only the $l = 1$ contribution remains through the integration over φ in

Eq.(16) . The subsequent integration over r considering the approximation of Eqs.(13,14) and taking an infrared cutoff ξ gives

$$A_{+,+} = \frac{2\pi i}{e} \int_R^{\xi} dr\ J_1\left(Q\,r\right)\ . \tag{18}$$

The first stage following the string formation corresponds to the friction regime at the beginning of which the correlation length is $\xi = \frac{1}{\lambda\eta}$. The total cross-section per unit length can be then computed numerically giving the results of Fig.1 . For large enough $\lambda^{-1} > 8$, and GUT scale $\eta = 10^{15}\ GeV$, the results can be fitted by

$$\frac{d\sigma_{AB}}{dL} = 1.86\,\xi\ + \frac{5.52}{p}\ , \tag{19}$$

showing that the ordinary Aharonov-Bohm behaviour of the second term becomes to be overrun by the cutoff contribution of the first one. It may be seen that in the friction regime, though the cone in the forward direction which gives rise to the cutoff is very relevant, the non forward contribution cannot be neglected.

For a final state with negative helicity, it turns out that $S^{(1)}_{-,+} = 0$ which could be expected since the Aharonov-Bohm scattering conserves the helicity [7]. On the other hand the cross-section for negative to negative helicity is $\backsim (\frac{M_o^2}{2p^2})^2$ smaller than Eq.(19) due to the fact that ν_R is essentially of positive helicity.

Regarding the contribution of the string Higgs field to the scattering of a positive to positive helicity ν_R , using Eqs.(8,9) ,

$$S^{(2)}_{+,+} = -i\frac{M_o}{2}\int d^4x\ \left\langle \mathbf{p}\acute{}, + \left|\psi^{\dagger}\sigma^2\psi^{\star}\left(1 - e^{i\varphi}f\ \right)\right| \mathbf{p}, + \right\rangle$$
$$= \frac{iM_oL}{2\mathcal{V}}\sqrt{\frac{\left(p_o\acute{} + p\acute{}\,\right)\left(p_o - p\right)}{2p_o\acute{}\,2p_o}}\,2\pi\delta\left(p_o\acute{} - p_o\right)\cos\left(\theta/2\right)\ \frac{\pi}{2p^2}\ \digamma_{+,+}\ , \tag{20}$$

where

$$\digamma_{+,+} = \int d\varphi\, dr\, r\, e^{-i\mathbf{Q}.\mathbf{r}}\left(1 - e^{i\varphi}\ f\,\right). \tag{21}$$

The use of the expansion Eq.(17) and the integration over φ leaves

$$\digamma_{+,+} = 2\pi\int dr\, r\ \left[J_0\left(Q\,r\right) + i\,f\ \ e^{i\beta}\,J_1\left(Q\,r\right)\right] = \frac{\pi}{2p^2}\Xi\ , \tag{22}$$

Figure 1: Contribution of the gauge field of the string to the $+\to+$ helicity scattering cross section in the friction regime for different values of λ. The lines represent the fit using Eq.(19).

Figure 2: Contribution of the Higgs field of the string to the $+\to-$ helicity scattering cross-section in the friction regime for different values of λ. The lines represent the approximation of Eq.(30).

where, together with the aproximation Eqs.(13,14) and cutoff ξ ,

$$\Xi = \int_o^{2p\xi} dz\, z\, J_0\left(z\,\cos\left(\theta/2\right)\right) + i\, e^{i\beta} \int_{2pR}^{2p\xi} dz\, z\, J_1\left(z\,\cos\left(\theta/2\right)\right)$$
$$= \Xi_o + i\,\Xi_1 e^{i\beta}\ . \tag{23}$$

With change from positive to negative helicity the calculation is analogous giving

$$S^{(2)}_{-,+} = i\frac{M_o L}{2\mathcal{V}}\sqrt{\frac{\left(p_o' - p'\right)\left(p_o - p\right)}{2p_o'\, 2p_o}}\ 2\pi\delta\left(p_o' - p_o\right)\sin\left(\theta/2\right)\ \frac{\pi}{2p^2}\ \Xi\ . \tag{24}$$

For the matrix element of $S^{(3)}$ without change of helicity, again using Eqs.(8,9),

$$S^{(3)}_{+,+} = -i\frac{M_o}{2}\int d^4x\ \left\langle \mathbf{p}', + \left|\psi^T\sigma^2\psi^\star\left(1 - e^{-i\varphi}\ f\ \right)\right| \mathbf{p}, +\right\rangle$$
$$= \frac{iM_o L}{2\mathcal{V}}\sqrt{\frac{\left(p_o' - p'\right)\left(p_o + p\right)}{2p_o'\, 2p_o}}\ 2\pi\delta\left(p_o' - p_o\right)\cos\left(\theta/2\right)\ \mathcal{G}_{+,+}\ , \tag{25}$$

where now

$$\mathcal{G}_{+,+} = \int d\varphi\, dr\, r\, e^{-i\mathbf{Q}\cdot\mathbf{r}}\left(1 - e^{-i\varphi}\ f\ \right). \tag{26}$$

With the same steps as above one gets

$$\mathcal{G}_{+,+} = 2\pi\int dr\, r\ \left[J_0\left(Q\,r\right) + i\,f\ e^{-i\beta} J_1\left(Q\,r\right)\right]$$
$$= \frac{\pi}{2p^2}\left(\Xi_o + ie^{-i\beta}\,\Xi_1\right)\ . \tag{27}$$

Analogously, for the change of helicity

$$S_{-,+}^{(3)} = \frac{iM_o L}{2\mathcal{V}} \sqrt{\frac{(p_o' + p')\,(p_o + p)}{2p_o'\,2p_o}} 2\pi\delta\,(p_o' - p_o)\sin(\theta/2)$$
$$\times \frac{\pi}{2p^2}\left(\Xi_o + ie^{-i\beta}\,\Xi_1\right)\ . \tag{28}$$

Using these results

$$\left|S_{+,+}^{(2)} + S_{+,+}^{(3)}\right|^2 \backsim \left(\frac{M_o}{p}\right)^2 \left|S_{-,+}^{(2)} + S_{-,+}^{(3)}\right|^2 , \tag{29}$$

indicating the fact that violation of helicity is favoured by the Majorana coupling.

The numerical computation of this dominant cross-section is shown in Fig.2 and can be fitted for $\lambda^{-1} \gtrsim 8$ by the approximate behaviour for $p\xi \gg 1$

$$\frac{d\sigma_{\mathcal{H}}}{dL} = 1.04\,\xi \left(\frac{M_o}{p}\right)^2 [1 + 0.48\ln(p\xi)]\ , \tag{30}$$

where the additional logarithmic dependence on the cutoff is a consequence of the phase in its interaction with the fermion as seen in Eq.(10).

For a later time after their formation, strings may reach [8] the scaling regime, due to a correlation length $\xi \backsim t$, when the universe cooled below $T_{sc} \simeq \frac{T_{GUT}^2}{M_{pl}} \backsim 10^{11} GeV$. This occurred for the time $t \backsim 10^{-28}s$, being the expansion of the universe scale due to radiation $a(t) \propto t^{1/2}$. Therefore for momentum $p > M_o$, $\xi p \gg 1$ with $\xi \gg R$ so that now the cutoff contribution dominates clearly over the ordinary Aharonov-Bohm term giving as approximation for the cross-section caused by the gauge field

$$\frac{d\sigma_{AB}}{dL} \simeq 2\left(1 + \frac{M_o^2}{4p^2}\right)^2 \xi\ , \tag{31}$$

and obviously the contribution given by the string Higgs field is even better approximated by Eq.(30) in the scaling regime. Since the correlation length is much larger in the scaling regime than in the friction one, the above cross-sections are correspondingly larger in the former case.

4 Capture of fermions by strings with emission of bosons

This process has analogy with the capture of an electron by a nucleus with the emission of a photon, where the description is given in terms of the interaction of the electron with the quantized radiation field in addition to the Coulomb attraction.

Therefore we add the quantum fluctuations to the classical configurations of the string Higgs and gauge fields as

$$\Phi = \Phi^{cl} + \widehat{\Phi} \qquad , \qquad \mathcal{A}_\mu = \mathcal{A}_\mu^{cl} + \widehat{\mathcal{A}}_\mu \ . \tag{32}$$

Thus we will have as interaction with the additional quantum boson fields

$$\mathcal{L}^{\text{quan}} = -\frac{ie}{2}\psi^\dagger \sigma^\mu \, \widehat{\mathcal{A}}_\mu \psi - \frac{ig}{2}\left(\psi^\dagger \widehat{\Phi} \psi^C - \psi^{C\ \dagger} \widehat{\Phi}^\dagger \psi\right) \ , \tag{33}$$

for which the conditions for the validity of the perturbation treatment are $e \lesssim 1$ and $g \lesssim 1$.

Now for the fermion field we must consider the free solutions and the zero-mode states which can be formed with the background of the classical string configuration, i.e.

$$\psi = \widehat{\psi}_{free} + \widehat{\psi}_{zm} \ , \tag{34}$$

where $\widehat{\psi}_{free}$ is given by Eq. (4) as before whereas the zero mode term will be

$$\widehat{\psi}_{zm} = \sum_{p_z > 0} \left[c_o\,(p_z, +)\, \mathcal{U}_o\,(p_z, +)\; e^{-i\omega t} + c_o^\dagger\,(p_z, +)\, \mathcal{U}_o^\star\,(p_z, +)\; e^{\,i\omega t} \right] \ , \tag{35}$$

with $\omega = p_z$, describing massless particles which move along the positive z-axis through the anticommuting operators c_o . The zero-mode wave funtion is given [9] by

$$\mathcal{U}_o\,(p_z, +) = \frac{\widetilde{M}}{\sqrt{2\pi L}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp\left(-\int_0^\rho \left[\frac{M_o}{M_{\mathcal{H}}} f\ (\rho') + \frac{a\,(\rho')}{2\rho'} \right] d\rho' \right) \exp\,(ip_z\, z) \ , \tag{36}$$

where $\widetilde{M}^{-1}$ gives its effective radius, $\acute{\rho} = M_{\mathcal{H}}\, r$ and the normalization is $\int d^3x\, |\mathcal{U}_o\,(p_z, +)|^2 = 1/2$. It is clear that this requires a more detailed description of the classical fields inside the string than that given by Eqs.(13,14), i.e. [10]

$$f\,(\acute{\rho}) = f_o\, \rho\acute{} \qquad , \qquad a\,(\acute{\rho}) = a_o\, \acute{\rho}^2 \qquad , \ \rho\acute{} < 1 \ , \tag{37}$$

a_o and f_o being constants that, from the normalization condition, give $\widetilde{M} = M_{\mathcal{H}} \sqrt{\frac{M_o}{M_{\mathcal{H}}} f_o + \frac{a_o}{2}}$. The boson quantum fields outside the string are massive and are described in terms of operators with usual commutators for the complex Higgs and real gauge ones as

$$\widehat{\Phi}(x) = \sum_{\overrightarrow{k}} \frac{1}{\sqrt{2k_o \mathcal{V}}} \left(a_k e^{-ik.x} + b_k^\dagger e^{ik.x} \right) \ , \tag{38}$$

$$\widehat{\mathcal{A}}_\mu(x) = \sum_\lambda \sum_{\mathbf{p}} \frac{1}{\sqrt{2p_o \mathcal{V}}} \left(\varepsilon_\mu(p,\lambda)\, a(p,\lambda) e^{-ip.x} + \varepsilon^\star_\mu(p,\lambda)\, a^\dagger(p,\lambda) e^{ip.x} \right) \ , \tag{39}$$

the polarization vectors satisfying

$$\varepsilon^\star_\mu(p,\lambda)\varepsilon^\mu(p,\lambda\acute{}) = -\delta_{\lambda\lambda\acute{}} \qquad , \qquad \sum_\lambda \varepsilon_\mu(p,\lambda)\varepsilon^\star_\nu(p,\lambda) = -\, g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2_{\mathcal{A}}} \ . \tag{40}$$

The capture of a neutrino with emission of a superheavy Higgs particle $\nu_R \to \nu_{zm} + \Phi$ is produced by the second term of Eq. (33) giving at first order of perturbation theory the probability amplitude

$$\mathcal{S}_{zm\,\Phi,\nu_R} = \frac{g}{4} \frac{\widetilde{M}}{\mathcal{V}\sqrt{\pi q_o L}} \sqrt{\frac{p_o + p}{2p_o}} 2\pi\delta\,(p_o - q_o\acute{} - q_o)\, 2\pi\delta\,(q_z\acute{} + q_z)\ \Gamma\,(Q) \ , \tag{41}$$

where

$$\Gamma\,(Q) = \frac{1}{\sqrt{2}} \int d^2x\, e^{i\mathbf{x}_T.\mathbf{Q}} \exp\left(- \int_0^\rho \left[\frac{M_o}{M_{\mathcal{H}}} f\ (\rho\acute{}\,) + \frac{a(\rho\acute{}\,)}{2\rho\acute{}} \right] d\rho\acute{} \right) \ . \tag{42}$$

This integral in the transverse plane of the string, with the momentum transferred to it $\mathbf{Q} = \mathbf{p} - \mathbf{q}_T\acute{}$ can be calculated approximately [9] expanding the plane wave in Bessel functions to give

$$\int d\varphi\ e^{i\mathbf{x}_T.\mathbf{Q}} = 2\pi\, J_0\,(Q\, r) \ , \tag{43}$$

Figure 3: Comparison of the cross sections of capture of ν_R to form zero modes with emission either of a Higgs or a vector massive boson .

which together with the form of the classical fields inside the string Eq.(37), being neglegible their contribution outside it, allows to obtain

$$\Gamma(Q) \simeq \sqrt{2}\pi \frac{1}{\widetilde{M}^2} \exp\left(-\frac{Q^2}{2\widetilde{M}^2}\right) \ . \tag{44}$$

The numerical evaluation of the capture total cross-section is shown in Fig.3 taking all the masses of the same order.

Regarding the capture with the emission of one gauge vector boson $\nu_R \rightarrow \nu_{zm} + \mathcal{A}$, the first term of Eq. (33) gives the probability amplitude

$$\begin{aligned} \mathcal{S}_{zm\,\mathcal{A},\nu_R} = & \frac{ie}{2\mathcal{V}} \frac{\widetilde{M}}{\sqrt{\pi L}} \sqrt{\frac{p_o + p}{2p_o 2k_o}} 2\pi\delta\left(p_o - k_o - q_o\right) 2\pi\delta\left(q_z + k_z\right) \\ & \times\ \chi_o^\dagger \sigma^\mu \varepsilon_\mu^\star(k,\lambda)\, \chi\left(\mathbf{p},+\right) \Gamma\left(Q\right) \ , \end{aligned} \tag{45}$$

where now $Q = |\mathbf{p} - \mathbf{k}_T|$, $\chi_o = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\chi\left(\mathbf{p},+\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\Gamma\left(Q\right)$ is that of Eq.(42) .

The numerical results for the total cross-section with the same approximation of equal masses and taking [11] $\alpha_{GUT}^{-1} = \frac{4\pi}{e^2} = 26$ are also presented in Fig. 3 .

One sees that both cross-sections show a resonant behaviour for values of the momentum of ν_R of the order of its mass .

5 Cosmological implications and conclusions

We have analysed the possibility that fermions that acquired mass in the GUT epoch of the universe evolution could have been captured by cosmic strings formed by the breaking of an Abelian subgroup at this scale.

In the case that the GUT symmetry that contained this subgroup corresponded to $SO(10)$, the fermion to be considered is the ν_R . These neutrinos captured by the string would produce a superconducting current, even though they are neutral, in the sense that inside it they travel at the velocity of light.

This current stabilizes closed strings which otherwise would contract and disappear. The superconducting microscopic loops, vortons, detach from the string dynamics and behave as a quasi-stable dark matter that might have survived till the present condensed in the halo of our galaxy. Since vortons have a small probability of quantum decay by tunneling of their constituents, they might be the origin of the ultra-high energy cosmic rays (UHECR) which have been observed without identification of their astrophysical sources [12] .

To see which is the flux of UHECR produced by vortons it is necessary to estimate their density whose evolution with the temperature of universe T starting from their formation at T_f will be

$$n_v(T) = n(T_f)\left(\frac{T}{T_f}\right)^3 , \tag{46}$$

being considered [13] that $n(T_f) \backsim (\xi(T_f))^{-3}$.

During the friction regime an estimation [14] is

$$\xi^{\,fr}(T) \simeq (m_{PL})^{1/2}\frac{T_{GUT}}{T^{\,5/2}} , \tag{47}$$

where m_{PL} is the Plank mass, so that for $T_f = T_{GUT}$ from Eq.(46) $n_v^{fr}(T) \simeq 10^{-6}\,T^3$, whereas for the formation at the end of this period $T_f \simeq 10^{11}GeV$ $n_v^{fr}(T) \simeq 10^{-24}\,T^3$.

Regarding the number of fermionic carriers in the loop [13]

$$N \simeq \xi(T_f)T_{GUT} , \tag{48}$$

in the friction regime $N^{\,fr} \backsim 100$ if $T_f = T_{GUT}$, and $N^{\,fr} \backsim 10^{12}$ at the end of it when $T_f \simeq 10^{11}GeV$.

Looking now at the formation in the scaling regime valid for $T \lesssim 10^{11}GeV$ where

$$\xi^{sc} \simeq H^{-1} \simeq \frac{m_{PL}}{T^2} , \tag{49}$$

being H the Hubble parameter, $n_v^{sc}(T) \lesssim 10^{-24}T^3$ and $N^{sc} > 10^{12}$, both in agreement with the limit of the friction epoch.

As a consequence the number of fermions in vortons per unit volume $N\,n_v(T)$ if incorporated at the beginning of friction regime is $10^{-4}T^3$, whereas if incorporated at the beginning of the scaling one will be $10^{-12}T^3$. Therefore the ratio of these incorporated fermions per unit comoving volume is 10^8 , which is equal to the inverse ratio of times of formation. A similar analysis for the vorton formation during the scaling regime indicates that the density of absorbed fermions per unit comoving volume goes like $t_f^{-1/2}$.

It is obvious that the above ratio of fermions equals the one of lengths of original closed strings which at formation will be $\xi\left(T_f\right) \times \xi^{-3}\left(T_f\right)$. Therefore, being the probability of capture per unit length independent on time, the formation of vortons should be equally probable in friction and scaling regimes. However if one includes the motion of the original strings, which is different in both regimes, the formation of vortons during the scaling period is less likely [15] .

Even without this last consideration, from the above estimation of vortons density it is clear that their relevance for UHECR will be much more important if they were formed at the beginning of the friction epoch.

The considered example of $SO(10)$ is the simplest one since ν_R is the only non-ordinary fermion and acquires mass at the GUT scale. In order to have exotic fermions electrically charged and which have zero modes giving way to superconducting currents in the common sense of the word, we should consider the unification of interactions under a larger group as E_6 . But in this case the addition of 11 fermions, apart from ν_R , would make the analysis of the problem considerably harder.

Regarding the scattering of fermions by straight and long strings one may note that to the traditional Aharonov-Bohm effect due to the gauge potential, as in the solenoid case, also the interaction with Higgs field which generates the fermion mass must be added. From our calculation, one sees that this effect in the total scattering cross-section increases with the separation among strings faster than that due to the gauge field by a logarithmic factor which is related to the winding phase present also at large distances.

For this kind of strings their density length will be $\backsim 1/\xi^2$ and, subtracting the universe expansion, the corresponding one per unit comoving volume in the scaling regime will go like $1/t$. Since the cross-section per unit string length for scattering of neutrinos increases at least as t , the effect due to the targets in unit comoving volume will be roughly constant.

To make the process of generation of superconducting currents more realistic, one should take into account the propagation of neutrinos in the plasma outside the string, the influence of the motion of the latter and the fluctuations of the field equivalent to the electric one which could produce jumps of the fermions from negative to positive energy inside the core.

References

[1] A. Vilenkin and E. P. S. Shellard, Cosmic Strings and Other Topological Defects, Cambridge University Press, Cambridge, 1994; M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. **55** (1995) 478

[2] M. G. Alford and F. Wilczek, Phys. Rev. Lett. **62** (1989) 1071

[3] W. B. Perkins, L.Perivolaropoulos, A. C. Davis, Robert H. Brandenberger, A. Matheson Nucl. Phys. B **353** (1991) 237 ; S. Sakoda and M. Omote, J. Math. Phys. **38** (1997) 716

[4] R. Brandenberger, A. C. Davis and A. M. Matheson, Nucl. Phys. **307** (1988) 909

[5] R. Jackiw and P. Rossi, Nucl. Phys. B **190** (1981) 681

[6] P. D. Mannheim, Int. Jour. of Theor. Phys. **23** (1984) 643

[7] F. Vera and I. Schmidt, Phys. Rev. D **42** (1990) 3591

[8] A. Albrecht and N. Turok, Phys. Rev. Lett. **54** (1985) 1868 , D. Bennett and F. Bouchet, Phys. Rev. Lett. **60** (1988) 257 , M. Hindmarsh, Phys. Rev. Lett. **80** (1998) 2277

[9] S. C. Davis, W. B. Perkins and A. C. Davis, Phys. Rev. D **62** (2000) 043503

[10] M. Hindmarsh and T. W. B. Kibble , Rept. Prog. Phys. **58** (1995) 477

[11] U. Amaldi, W. de Boer and H. Forstenau, Phys. Lett. B **260** (1991) 447

[12] L. Masperi and M. Orsaria, Astropart. Phys. **16** (2002) 411

[13] B. Carter and A. C. Davis , Phys. Rev. D **61** (2000)123501

[14] R. Brandenberger, B. Carter, A. C. Davis and M. Trodden , Phys. Rev. D **54** (1996) 6059

[15] C. Martins and E. Shellard, Phys. Rev. D **57** (1998) 173